

# Spooky Black Holes and Gravitomagnetism

# **Paolo Christillin**



Abstract: Wide spread misconceptions about the reality of space curvature in General Relativity (GR) and their relevance in connection to the existence of black holes are revisited. The mean life of proposed black holes is estimated and their practical non existence is emphasized. The repulsive effect of gravitational self energy is underlined. The relevance of gravitomagnetism to account for alleged black hole effects is stressed.

Keywords: Black holes, Gravitational Self Energy, Metric, Singularities.

# I. INTRODUCTION

 $\mathbf{T}$  he origin of the idea of black holes (BH) dates back, as well known, to Michell [1] and Laplace [2] and has received renewed interest with the advent of General Relativity (GR) [3]. The possible existence of these exotic objects has gained great popularity and in recent times experimental results [4], culminating in the Nobel Prize award [5] [6] seem to support their presence. This is accompanied by the theoretical analysis of [7].

However the identification of these objects with the observations represents an open problem as will be examined in the following sections. First, the relevance of the metric in this context will be analyzed. Second, the estimate of intermediate mass BH will be made along the lines of the Universe time dependent age. This strongly questions the previous identification. A proposed alternative, due to gravitomagnetism, is then presented.

## **II. THE METRIC**

The implementation of GR corresponds to the modification of the invariant Minkovski interval. This depends on the metric and for non-rotating uncharged objects, two main ones exist. The famous Schwarzschild and the Painleve'-Gullstrand (P-G) one.

Although "equivalent" (see below) their predictions for BH are however different and the aim of the present paper is to enquire what is real and what can be considered just a pathological feature of the first one. In addition, the physical motivations which privilege the P-G metric will be examined and underlined. The Schwarzschild (Ss) [8] solution of GR is given by the invariant interval

$$ds'^{2} = c^{2}(1 - v^{2}(r)/c^{2})dt^{2} - \frac{dr^{2}}{\left(1 - \frac{v^{2}}{c^{2}}\right)} - dx_{\perp}^{2} \dots (1)$$

Manuscript received on 23 August 2024 | Revised Manuscript received on 17 September 2024 | Manuscript Accepted on 15 October 2024 | Manuscript published on 30 October 2024. \*Correspondence Author(s)

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The prime referring to Ss. The effect of gravitation on the Special Relativity (SR) invariant interval  $ds^2 = c^2 dt^2 - c^2 dt^2$  $dr^2$  is represented by the quantity

$$\epsilon = \frac{v^2(r)}{c^2} = \frac{2GM}{c^2 r}$$
 ... (2)

The previous expression Eq.1) is popular because of the similarity with the corresponding one in SR, where space and time variables have a direct link with reality.

This is not so in GR where deformed space-time i.e. correcting the SR invariant interval does not necessarily imply deformed space.

Indeed, the usual expression that GR dilates spacial distances has to be supplemented by the phrase: in the Ss metric only (see Fig.1).

As a matter of fact alternative solutions exist, in particular for uncharged non rotating objects, the Painleve'- Gullstrand (P-G) [9], [10] one, based on propertime in Euclidean space, and the two have been shown to be equivalent to Ss thanks to an ad hoc time transformation.

However, by this procedure one does not appreciate the interesting features of this metric.

Indeed it has been shown [11] [12] that it can be obtained in terms of the Newton free fall law [13] and the equivalence principle only and thus to be independent from GR:

GR = SR + Newton + equivalence principle

Nevertheless P-G reproduces at the same time the so called "three crucial tests" of GR. It is formulated in terms of Euclidean space and proper time. The invariant interval in this metric reads

$$ds^{2} = c^{2}dt^{2} - (dr - v(r)dt)^{2} - dx_{\perp}^{2} =$$
  
=  $c^{2}\left(1 - \frac{v^{2}(r)}{c^{2}}\right)dt^{2} + 2v(r)dtdr - dr^{2} - dx_{\perp}^{2} \dots$  (3)

and the consideration of the radial part (dt = 0)  $dr^2$  confirms its Euclidean nature.

The catchy picture of a funnel-like space deformation (which seems to suggest an unexcapable fall of all objects towards the source, Fig.1) is simply replaced by the standard familiar Euclidean one i.e. the distance between two concentric spheres is simply the familiar one. Fig.2.

In connection with the "reality" of space deformation (non Euclidean geometries) it should be obvious that if more metrics exist it is at least funny to think that different people deform space differently! The physical advantages of the P-G metric have already been stressed in [11]. In addition, the two previous equations (1) (3), although



Retrieval Number:100.1/ijap.B105304021024 DOI:10.54105/ijap.B1053.04021024 Journal Website: www.ijap.latticescipub.com

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Figure 1: Space Time Gravitational Deformation [14] to Which, Because of the Separate Treatment of Space and time Coordinates Corresponds an Analogous Deformation in Space Coordinates in the Ss Metric. Indeed, the Distance Between Concentric Spheres of Areas  $4\pi r^2$  and  $4\pi (r + dr)^2$  is  $\sqrt{\frac{1}{1-2GM/r}} dr > dr$ i.e. greater than the same in Euclidean space.

equivalent thanks to the above-mentioned transformation, yield however different predictions for photon emission from BHs, which will be dealt with in the next paragraph and which will be paramount in assessing their nonexistence. As a matter of fact, the photon radial velocity in the Ss metric reads

$$\frac{dr}{dt} = \pm c \left( 1 - \frac{v^2(r)}{c^2} \right) \dots \quad (4)$$
  
and  
$$\frac{dr}{dt} = \pm c - v(r) \dots \quad (5)$$

in the P-G one.

The second one embodies the equivalence principle which is at the base of GR, whereas the first does not. This seems to privilege the P-G metric among "equivalent" ones not only for its simplicity but also for its closer connection with reality.

As well-known a singularity at the Ss radius denoted by  $\bar{R} = \frac{2GM}{c^2}$  only arises for the first metric. This is not totally unexpected. To put it bluntly it is as if in a problem e.g. with spherical symmetry one would use cylindrical coordinates and then be surprised at finding possible singularities. The same thing happens for the "warping" in the case of radiation, where a similar unfortunate choice of the metric convinced Einstein of the nonexistence of gravitational waves [16].

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Thus in the P-G metric the Ss singularity does not exist as neither does another one i.e. the fundamental one of gravitation at the Universe origin, cured by QM [17].



## Figure 2: In the P-G Metric the Distance Between two Concentric Spheres is Simply the Euclidean one. Thus, no Space Deformation Results. The Two-Dimensional Example is Trivially Translated to the Three-Dimensional Case

## III. UNFROZEN STARS AND THE SS METRIC

Let us turn now to consider the so-called frozen stars which is the word BHs were initially denoted by to explain their non observation. Their photon emission rate is dramatically different in the two previous metrics. In the Ss one [18] the expression of the radial photon velocity is

$$cdt = \frac{rdr}{\bar{R}-r}$$
 ... (6)

deriving from the previous expression  $dr/dt = c(1 - v^2/c^2)$ . It is clear that for r tending to  $\bar{R}$  the integral of the r.h.s. presents as the dominating contribution a logarithmic singularity which can be re-expressed as

$$r - \bar{R} \simeq e^{-\frac{c(t-t_0)}{2R}} \dots (7)$$

This results in a photon emission rate dominated by the preceding exponential, which for an estimated  $5M_s$  stellar core of  $\bar{R} \simeq 3$  km undergoing final collapse yields a characteristic time of  $10^{-4}s$ . Thus no photons emerge from the star after a timescale of some milliseconds. The physical interpretation of this result is immediate: since the funnel-like potential well becomes steeper and steeper approaching the Ss singularity at  $\bar{R}$ , photons are more and more hindered from escaping.

This effect is not present in the P-G metric, where

$$cdt = \frac{\sqrt{r}dr}{\sqrt{R} - \sqrt{r}} \dots (8)$$

In this case one would have

$$c(t - t_o) = r - \bar{R}$$
 ... (9)

and the presumed frozen star would emit as an ordinary one. Thus, the fact that we observe nothing has to be attributed to their nonexistence. Especially in that sense frozen stars are indeed equivalent to black holes. The pathology of the Ss metric is once more evident: the radial component of light velocity goes to zero at  $\overline{R}$  as in the P-G metric but with a different dependence on the potential. As mentioned, the latter *represents an implementation of the equivalence principle in the free-falling frames, where locally light is emitted isotropically in the very spirit of GR, making unnecessary its complicated machinery.* This is highlighted in Fig. 3a) where light distortion simply results thanks to the equivalence principle in the P-G metric.



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<sup>&</sup>lt;sup>1</sup>An interesting historical reconstruction can be found in [15]. The contribution of Painleve' can be appreciated not only for the mathematics (in the invariant GR interval two arbitrary functions of r appear with quite general requirements) but also for having introduced in Newtonian physics the notion of proper time. Indeed, the apparent puzzling coexistence of different metrics can be traced to the fact there is an infinity of r solutions meeting the Einstein conditions. Thus, GR does not uniquely determine the invariant interval and in that sense the above physical requirement of the P-G one puts it on a privileged status.



#### **IV. BLACK HOLES LIFETIME**

Having seen that a BH would radiate as an ordinary star, the fundamental issue is therefore to understand why it does not. We need a decisive argument. And that is given by the estimate of their mean life. The black hole lifetime can be obtained in two (equivalent for a black hole) alternative ways. The first is

$$t = \frac{R}{c} \dots (10)$$

i.e. the standard way the approximate age of the Universe is evaluated (a more realistic could be derived from Fig.3b). It can be accompanied by the alternative expression, which to the best of our knowledge has not been considered so far, from which an interesting result emerges, namely the Universe dependence on its mass content

$$t' = \frac{GM}{c^3} \quad \dots \quad (11)$$

This shows that matter is not conserved [19].

A Less Qualitative Argument Applies. Consider the free fall time from Infinity Under the Newtonian Potential. Then

$$\frac{dr}{dt} = \sqrt{\frac{2GM}{r}} \quad \dots \quad (12)$$

The time T is given by

$$T = \frac{2}{3\sqrt{\frac{R^3}{2GM}}} \dots (13)$$



Figure 3: Both for Attraction (Newton) a) and for Expansion (Hubble) b) the P-G metric Represents the Local Implementation of SR. In the Former Case the SR Light Cone at ∞ (r = 0) is Progressively Curved by The Newton Free Fall Contribution. This yields a (t,r) plot in Euclidean Space, which is Equivalent (in Addition to no Singularities) to the Popular Geodetic Description

<b>Black Hole Classification Class</b>	Appr. Mass	Radius in m.	Т	T ' in sec
Ultramassive	$10^9 - 10^{11} M_S$	≥ 1,000 <i>AU</i>	10 <sup>3</sup>	10 <sup>5</sup>
Supermassive	$10^6 - 10^9 M_S$	0.001400AU	$1 - 10^{5}$	$10 - 10^4$
Intermediate	$10^2 - 10^5 M_s$	106	10 <sup>-2</sup>	10 <sup>-3</sup>
Stellar	$2 - 150)M_s$	$3 \times 10^{4}$	10 <sup>-4</sup>	$10^{-5} - 10^{-3}$
Micro	up to $M_{M \text{ oon}}$	up to 10 <sup>-3</sup>	10 <sup>-11</sup>	10 <sup>-12</sup> .

where R stands for the radius of the source. This applies of course to any Gravitational Object and Specifically to the BH case, where by use of the BH relation

$$T_{BH} = \frac{4GM}{3c^3}$$
 ... (14)

Of course, only for BHs M and R are connected so that the free fall time is alternatively  $T \simeq \overline{R}/c$ . The result has an immediate interpretation and confirms the dimensional result of [19] of the mean life of a black hole. The above escape time

Retrieval Number:100.1/ijap.B105304021024 DOI:10.54105/ijap.B1053.04021024 Journal Website: www.ijap.latticescipub.com (= free fall time) is a mean life measure since it applies also when the escaping particle is a part of the object at its surface. It should be once more emphasized that all is based just on the Newtonian potential which is the only physical ingredient of the P-G metric and hence of GR.

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In addition, the previous dimensional argument can be justified also with sweeping arguments of which the most convincing is probably its successful application at the time range extremes to the primeval Planck one  $(t_P = R_P/c)$  and to the present Universe  $(t_U = R_U/c)$  for both of which

$$\epsilon = \frac{2GM}{c^2\bar{R}} = 1 \quad \dots \quad (15)$$

If t = t' (of course at this level the factor 2 is irrelevant) the object is a black hole, which is not the case for none of the following Table. Let us underline that this result does not come from the consideration of unknown dynamics at these extreme conditions but only from general arguments. The relevant point is, however, not so much that they do not fulfill the black hole requirement but that even in the most favorable cases the estimated lifetime is a fraction of a year, irrelevant at cosmological scales, and the lighter candidates are even more short lived. Thus, BHs should exist but cannot be seen: very reminiscent of dark energy! This renders all related speculations [20], [21] just academic. Consider finally as a Gedanken experiment a"double" Pound Rebka one.

Suppose that it is performed in ordinary conditions on the earth above and below ground level. It yields correctly the red shift for photons with obvious meaning of the symbols.

$$\frac{\omega}{\omega'} = 1 + \frac{2hg}{c^2} \quad \dots \quad (16)$$

Imagine now doing the same thing above and below a BH. It straightforward to obtain, by expanding as before in  $h/\bar{R}$  and exploiting the BH condition in the term  $1 - \frac{2GM}{R+h}$ 

$$\frac{\omega}{\omega'} = -1 \quad \dots \quad (17)$$

This result is astonishing for two reasons: first because it does not depend on the height h (always in the small h approximation) but, more importantly, because it would imply a negative energy for the lower lying photon. This is independent of the metric and questions once more the reality of black holes.

Nevertheless, if existing, their interaction with an external probe ( $r > \overline{R}$ ) would result in

$$\left(\frac{v}{c}\right)^2 = \frac{\bar{R}}{r} \dots (18)$$

i.e., contrary to the folklore swallowing, just that of an ordinary mass.

However, it is interesting to enquire into the very academical case of a farther collapse down to the hypothetical singularity, presumably through radiation. It has been stressed [23] that the BH condition corresponds to a complete cancellation of the "bare" mass due to the self-energy and the nonexistence of black holes has been also reiterated by explicit calculations [24]. With excessive words of caution since that had been assessed to apply only to Newtonian black holes, whereas, for the previous arguments, it has a general validity. How can this be reconciled with the previous considerations? Simply by introducing the self-energy effect in the interaction i.e.

$$\left(\frac{v}{c}\right)^2 = \frac{\bar{R}}{r(1-\frac{\bar{R}}{r})}$$
 ... (19)

The repulsive effect of gravity at short distances is evident (as well as the free fall zero velocity where nothing is left to exert attraction). This closely parallels what had already been found in the treatment of the Universe expansion [17] and confirms in a more rigorous way the original naive approach [23]. The ensuing academic free fall law modification would not influence the three GR crucial tests, being an  $O(\epsilon^2)$  effect. We then plot as a function of  $\overline{R}$  the effect on an external probe at a given r.

#### V. THE CONTRIBUTION OF GRAVITOMAGNETISM

Let us then turn to a possible physical explanation of the observations. First of all our galaxy case presents a striking similarity with the M33 case, where the possible effect of gravitomagnetism [19] has been considered against MOND [22]



Figure 4: Velocity of an External Probe at A Given Distance Under the Attraction of a Classical Black Hole (Blue Line) and Taking into Account also its Self-Energy (Yellow Line) as A Function of the BH Radius. In the Second Realistic Case the Free Fall Velocity Deviates More and More for Decreasing Radii and Becomes Zero at the Origin, the Presumed Singularity Being Cured Just by Gravity

and the equivalent missing mass hypothesis. Indeed, one has, in this case, outskirts ( $r \simeq 10^{20}$  m) velocities of the order of 200 – 250 km/h whereas the ordinary Newtonian prediction yields something of the order of 150. The comparison between  $GM/r^2$  and the long range gravitomagnetic relativistic  $(1/c^2)$  field

$$h = \frac{GM}{c^2 r} \omega \quad \dots \quad (20)$$

( $\omega$  being the angular velocity of the core contributing to the acceleration as 2hv, v standing for the orbital velocity of the outskirts), shows them to be of the same order for

 $\omega = \frac{c^2}{rm}$ .

rv ·



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For the given r value, it yields a rotation period T of the order of 10<sup>8</sup> seconds much smaller than that of the orbiting particles  $T \simeq 10^{15}$  s.

This seems reasonable in line with our picture of the dynamics of galaxies rotation curves, even if, of course, it would need an experimental confirmation. More than that, because of the previous factor

$$\frac{GM}{c^2r}$$
 ... (21)

of the same form as that of a black hole, this gravitomagnetic term might counterfeit a black hole in the sense that a very massive rotating mass would be indistinguishable from it.

In particular the 1/r behaviour of gravitomagnetism provides the density  $\rho \simeq 1/r^2$  which disposes of missing mass. Thus, gravitomagnetism enters on the same footing of Newton's law in describing long range gravitational effects.

## VI. CONCLUSIONS

The aim of the present work has been twofold. First it has been shown that the presumed existence of BHs is present only in the Ss metric whose criticalities have been stressed. Second it has been shown that long range gravitomagnetism, already successful in disproving MOND and dark matter, might account for their observation. This could be confirmed if accompanied by the observation of their rotation. In conclusion it appears that the present trend of physics is, alas, more of philological nature, namely of asserting the existence of non-existing entities and labelling them with catchy words: missing mass, dark energy, black holes, space warping, down to quintessence and to define a discrepancy a tension and a "dull" tide effect a spaghettification. This is accompanied by the addition of ad hoc add ons [25]. In conclusion, contrary to Thorne's opinion ("Einstein himself wrote a regrettable paper arguing that black holes cannot exist"), we agree with Einstein, although for different reasons, about the nonexistence of black holes.

### ACKNOWLEDGMENTS

I wish to thank P. Amato, L. Bonci and E. Cataldo (ABC) for continuous support and encouragement and for a critical reading of the manuscript, A. Rucci for a figure and in particular L. Bonci for many stimulating discussions.

## **DECLARATION STATEMENT**

I must verify the accuracy of the following information as the article's author.

- Conflicts of Interest/ Competing Interests: Based on my understanding, this article has no conflicts of interest.
- Funding Support: This article has not been sponsored or funded by any organization or agency. The independence of this research is a crucial factor in affirming its impartiality, as it has been conducted without any external sway.
- Ethical Approval and Consent to Participate: The data provided in this article is exempt from the requirement for ethical approval or participant consent.

- Data Access Statement and Material Availability: The adequate resources of this article are publicly accessible.
- Authors Contributions: The authorship of this article is contributed solely.

#### REFERENCES

- 1. Michell J., In a Letter to Henry Cavendish, Esq. F.R.S. and A. S. , 1783
- 2. Laplace P. S., Traite' de Me'canique ce'leste, Paris, Charles Crapelet, (1799)
- 3. Einstein A., , Die Grundlage der allgemeinen Relativit atstheorie , 1916, der Physik. 49. Annalen S. 769 822. https://doi.org/10.1002/andp.19163540702

Wikipedia, Black holes 4.

- 5. Ghez, A. M.; Klein, B. L.; Morris, M.; et al. (1998). "High Proper-Motion Stars in the Vicinity of Sagittarius A\*: Evidence for a Supermassive Black Hole at the Center of our Galaxy". The Astrophysical Journal 509 (2) 678-688, arXiv astro-ph/9807210. Bibcode: 1998ApJ 509 678 G. doi:10.1086/306528. S2CID 18243528. https://doi.org/10.1086/306528
- Eckart, A; Genzel, R. (1996) " observations of stellar proper motions 6. near the Galactic Centre" Nature 383 (6599) 415 Bibcode:1996Natur.383..415E. doi:10.1038/383415a0. S2CID 4285760. https://doi.org/10.1038/383415a0
- 7. M.Begelman, M. Rees, Gravity's s fatal attraction, W.H. Freeman and Company, New York, 1996
- 8. Schwarzschild, K. (1916). "Uber das Gravitationsfeld eines Massenpunktes" nach der Einsteinschen Theorie". Sitzungsberichte der K"oniglich Preussischen Akademie der Wissenschaften. 7: 189 196.
- 9. Painleve' P., La mecanique classique et la theorie de la relativite' C. R. Acad. Sci. (Paris) 1777 680 (1921)
- 10. Gullstrand A., Allgemeine loesung des statischen Einkoerper-Problems in der Einsteinschen Gravitationstheorie, Arkiv. Mat. Astron. Fys. 16(8) 1 15 (1922).
- Christillin P., Morchio.G arXiv:1707.05187v2 [gr-qc] 3 Oct 2019 11.
- Trails in Modern Theoretical and Mathematical Physics, Springer, 12. ISBN 978-3031-44988-8 (eBook) https://doi.org/10.1007/978-3-031-44988-8 https://doi.org/10.1007/978-3-031-44988-8
- 13. Newton I., Philosophiae Naturalis Principia Mathematica, London, 1687; Cambridge, London, 1713: 1726 https://doi.org/10.5479/sil.52126.39088015628399 14.
- Wikipedia, Space curvature, CNX-UPhysics-13-07
- 15. Fric J. (2014): Painleve', une contribution trop originale a la relativite' generale pour avoir ete' comprise а l'epoque. https://journals.openedition.org/bibnum/851 https://doi.org/10.4000/bibnum.851
- Christillin P., Journal of Modern Physics, 12, 798-805. (2021) 16. https://doi.org/10.4236/jmp.2021.126051
- 17. Christillin P., Journal of Modern Physics, 12, 806-828. (2021) https://doi.org/10.4236/jmp.2021.126052
- 18. Maoz D., Astrophysical in a nutshell, Princeton University Press, 2007 Christillin P., Journal of Modern Physics, 14, 18-30. (2023). 19. https://doi.org/10.4236/jmp.2023.141002
- 20. Hawking, S.W. (1975)"Particle creation by black holes" Communications in Mathematical Physics. 43 (3) 199-220 Bibcode:1975CMaPh..43..199H. doi:10.1007/BF02345020. S2CID 55539246. https://doi.org/10.1007/BF02345020
- Beckenstein, A. (1972). "Black holes and the second law". Lettere al 21 Nuovo Cimento. 4(15) :99-104 doi:10.1007/BF02757029. S2CID 120254309. https://doi.org/10.1007/BF02757029
- 22. 365 370, 1983 Milgrom, M. Astrophysical Journal 270 Bibcode:1983ApJ...270..365M. doi:10.1086/161130. https://doi.org/10.1086/161130
- 23. Christillin Р., EPJP. 126, 48. (2011)https://doi.org/10.1140/epjp/i2011-11048-2
- 24. Dillon G., arXiv: 1303.2577v1 [physics.gen-ph] 15 Feb. 2013
- Today, 25. Perlmutter S. Physics 53 (2003)56. https://doi.org/10.1063/1.1580050



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# **Spooky Black Holes and Gravitomagnetism**

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